

Ecosizer – Central Heat Pump Water Heating Sizing Tool



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1 SIZING

This section describes the mathematical and physical principles behind the sizing of domestic hot water (DHW) systems in EcosizerEngine. Sizing determines the minimum heating capacity (kBTU/hr) and minimum primary storage volume (gallons) that a heat pump water heater (HPWH) system must have to meet the building's peak daily hot water demand. Two sizing paths exist: normal sizing (where the primary DHW heater is allowed to run at any time of the day) and load-shift sizing (the heater is prohibited from running during a user-defined shed window).

1.1 NORMAL SIZING OVERVIEW

Normal sizing assumes the HPWH can run during any hour of the day and must deliver the full daily DHW load within a designer-specified maximum runtime in hours per day, H (default 16 hours). The procedure follows three steps:

- 1) **Step 1 – Required heating capacity.** The HPWH must heat enough water to meet the entire day's demand within the allotted run time. V_d , the total daily DHW demand at supply temperature (gal), is based on the user input magnitude, n , and calculated using Table 1 based on the type of building the DHW system serves. Values for non-multifamily buildings are derived from ASHRAE Application Handbook 50.19 Figure 24. Multifamily buildings calculated gallons per day per person (gpdpp) using user inputs.

Building Type	Units for n	V_d
Men's Dorm	Number of Students	$23.6 * n$
Women's Dorm	Number of Students	$19.6 * n$
Motel	Number of Rooms	$21.4 * n$
Nursing Homes	Number of Beds	$23.4 * n$
Full-Meal Restaurants	Maximum Meals per Hour	$11.032 * n$
Drive-ins, Snack Shops	Maximum Meals per Hour	$6.44 * n$

Elementary Schools	Number of Students	$1.24 * n$
Junior High Schools	Number of Students	$3.75 * n$
Senior High Schools	Number of Students	$3.26 * n$
Office Buildings	Number of People	$2.1 * n$
Multifamily Building	Number of People	$gpdpp^* * n$

Table 1. Daily hot water usage magnitude by building type.

* User input or calculated gallons per day per person

The required heating capacity in BTU/hr is:

$$Q = \frac{V_d}{H} \times \rho c_p \times \frac{(T_{supply} - T_{CW})}{D}$$

where H is the maximum daily run time (hours), T_{supply} is the supply temperature (°F), T_{CW} is the design cold-water inlet temperature (°F), D is the defrost derate factor (default 1.0, dimensionless¹), and $\rho c_p = 8.354 \text{ BTU}/(\text{gal} \cdot ^\circ\text{F})$ is the volumetric heat capacity of water.

- 2) **Step 2 – Running volume (supply temperature).** The running volume is the area of the storage tank between the aquastat fraction (the location of the ‘on’ setpoint on the storage tank) and the outlet of the tank that sends hot water to the building or next piece of the system.

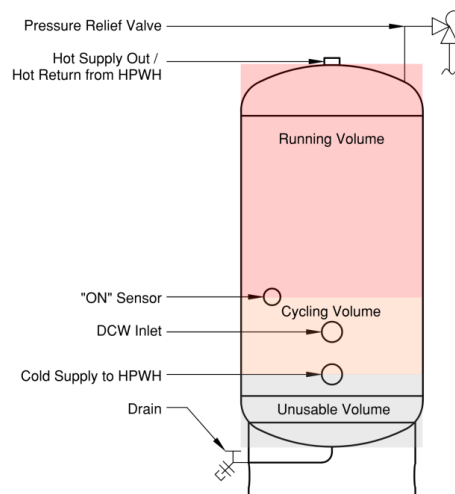


Figure 1. Diagram of a Storage Tank with Running Volume.

To account for the worst case scenario, the size of running volume is the amount of usable hot water that must be held in storage at the start of a peak demand period so that supply never runs out before the HPWH can replenish it. This is found using a 24 hour load shape for the design day (or average day when sizing for load shifting) of the building the DHW system will serve. The 24-hour load shape, $L(h)$,

¹ Defrost derate is set to 1.0 assuming the engineer applies the defrost derate, specific to their chosen equipment, to the result.

where $L(h)$ is the gallons of DHW at T_{supply} consumed by the building during hour $h \in \{0, 1, 2, \dots, 23\}$, is compared hour-by-hour against G , the uniform generation rate of water at supply temperature needed to serve the building:

$$G = \frac{V_d}{H}$$

The hourly difference for hour h is $d(h) = G - L(h)$. Positive values are surplus (heater catching up); negative values are deficit (demand exceeding current generation, thus requiring previously heated DHW from the storage tank to meet the load during h). To handle demand periods that straddle midnight, the algorithm calculates $d(h)$ over two consecutive days.

Over each hour h^* where d transitions from positive to negative (a surplus to deficit transition), V_{run} is the maximum cumulative deficit:

$$V_{run} = \max_{h^*} \left\{ -\min_{k \geq 0} \left(\sum_{j=0}^k d(h^* + j) \right) \right\}$$

If no such deficit exists (generation rate exceeds peak hourly demand every hour), $V_{run} = 0$ and no storage is needed (this is the physical minimum of the sizing curve), however, if a deficit exists, V_{run} is the size of the required thermal battery of DHW storage at T_s for the system to get through a peak demand day without a DHW outage. Figure 2 illustrates finding V_{run} .



Figure 2. Finding V_{run} from a Multifamily Building Load Shape.

This figure shows each possible candidate for V_{run} starting at the beginning of each peak period, h^* . In this case, the largest required volume, or largest positive area between $L(h)$ and G is the top right candidate.

- 3) **Step 3 – Physical storage volume (storage temperature).** The running volume is expressed at supply temperature, meaning the storage tank needs to be able to store enough water to produce V_{run} gallons

of supply temperature water while the tank is recharging. The physical tank stores water at the higher storage temperature, T_{st} , and is not perfectly stratified; there is a warm transition zone between the cold inlet layer at the bottom and the fully-hot zone at the top. Only a fraction of the physical tank volume is usable. The stratification factor at the point the heater is triggered ON, $S(f_{on}, T_{on})$ (described in Section 1.2), captures this fraction of usable volume:

$$V_{storage} = \frac{V_{run}}{S(f_{on}, T_{on})}$$

1.2 STRATIFICATION FACTOR

The stratification factor, $S(f, T)$, is the ratio of the supply-temperature gallons that a real (imperfectly stratified) 100-gallon tank can deliver (at a specified time) divided by the total number of gallons in the tank. Given a fraction of tank height, f , and a temperature of water, T , between T_{CW} and $T_{storage}$ at that fraction of tank height, $S(f, T)$ is calculated by finding how many gallons of T_{supply} temperature water can be produced by the portion of the tank that is hot and unusable.

The tank temperature profile is modelled as a linear gradient in the transition zone between sections of T_{CW} and $T_{storage}$ temperature water. To explain this, let us calculate the stratification factor at the point the heater is triggered ON, $S(f_{on}, T_{on})$. f_{on} is the fractional height of the ON setpoint's temperature sensor (0.0 = bottom, 1.0 = top) and T_{on} is the setpoint temperature for the ON setpoint. m is the stratification slope (in °F per 1% of tank height, found through PG&E lab data – see Table 2 for values of m calculated through lab data).

System Configuration	Stratification Slope m
Multi-Pass Return To Primary	0.8 °F per 1% of tank height
Single-Pass Return To Primary	1.7 °F per 1% of tank height
Swing Tank, Parallel Loop Tank, No Recirc	2.8 °F per 1% of tank height

Table 2. Stratification Slopes by System Configuration Type.

Derived from Ecotope's collected lab data on storage tank temperatures

The temperature at any fractional tank height f (where $0 \leq f \leq 1.0$) in the tank at this moment is:

$$T(f) = \min\left(T_{storage}, \max(T_{CW}, T_{on} + (f - f_{on}) * 100 * m)\right)$$

The heights at which T equals supply temperature T_{supply} (lower boundary of usable zone) and storage temperature $T_{storage}$ (upper boundary) are:

$$f_{supply} = \max\left(f_{on} + \frac{(T_{supply} - T_{on})}{m * 100}, 0.0\right)$$

$$f_{storage} = \min\left(f_{on} + \frac{(T_{storage} - T_{on})}{m * 100}, 1.0\right)$$

The number of T_{supply} temperature gallons producible from the 100-gallon tank in this moment, $V_{supply, strat}$, is found by integrating over the two usable zones — the transition zone $[f_{supply}, f_{storage}]$ and the fully-hot zone $[f_{storage}, 1.0]$:

$$V_{supply, strat} = \int_{f_{supply}}^{f_{storage}} \frac{(T(f) - T_{CW})}{(T_{supply} - T_{CW})} df + (100 - V_{sub storage}) \times \frac{(T_{storage} - T_{CW})}{(T_{supply} - T_{CW})}$$

Where $V_{sub storage}$ is the volume of water in the 100 gallon tank below storage temperature such that $V_{sub storage} = f_{storage} * 100$. The stratification factor at the point the heater turns ON is

$$S(f_{on}, T_{on}) = \frac{V_{supply, strat}}{100 \text{ gallons}}.$$

Modeled Stratification

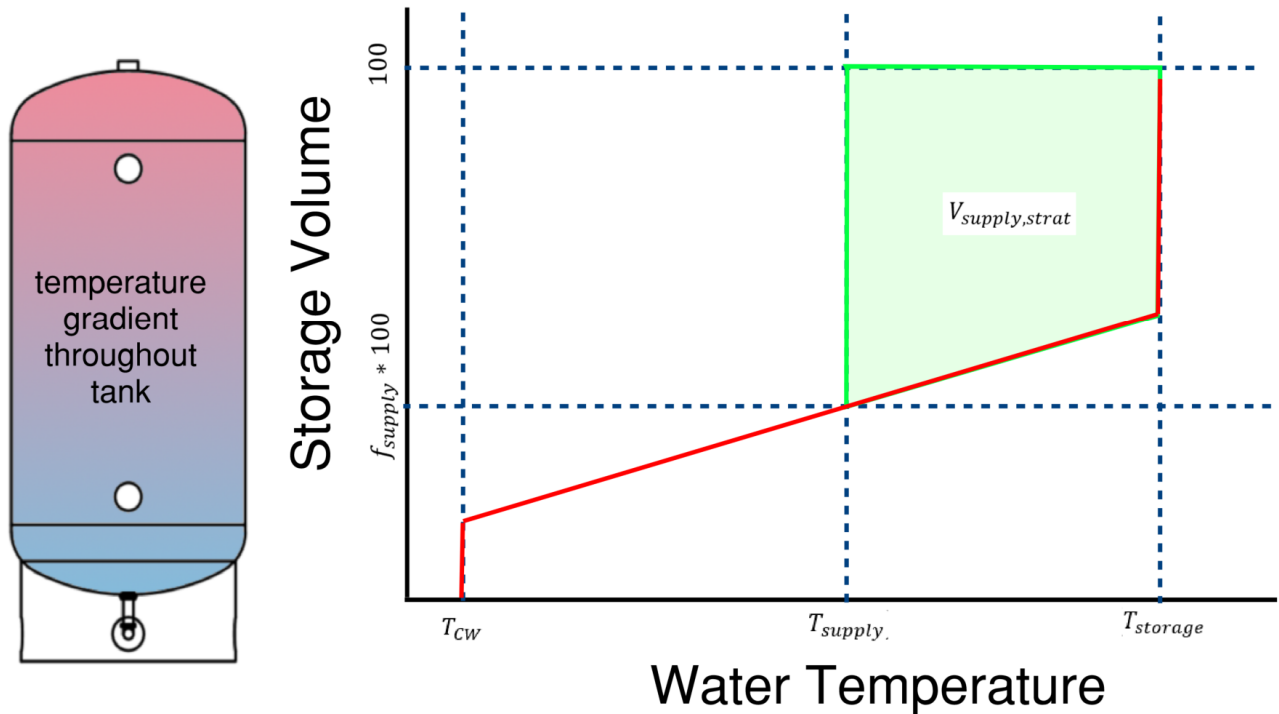


Figure 3. Finding $V_{supply, strat}$

This figure shows the area of the storage tank that is above supply temperature water. The green zone on the graph represents the number of gallons of supply temperature water that can be produced by water at each tank height

1.3 SIZING CURVE

The sizing curve is a design tool showing the capacity–storage trade-off for a given building. It is generated by sweeping H from 24 hours down to the physical minimum (where G is peak hourly demand, thus $V_{run} \rightarrow 0$) in

0.25-hour steps, computing Q and $V_{storage}$ at each step. The default design point corresponds to the default value of H for the system type being sized for (16 hours for most system types). Moving left on the curve (smaller storage) requires higher capacity; moving right (more storage) permits lower capacity.

1.4 LOAD-SHIFT SIZING

“Load shifting” for DHW heating is when the heater is discouraged from running during a shed period (a window of time such as 4 PM – 9 PM) to avoid drawing power during certain peak demand periods on the electrical grid. The system must pre-heat and store enough water during the preceding load-up period (the period directly before a shed period) to adequately serve DHW through the shed period. When sizing for a load-shifting system, the load-shift sizing runs alongside normal sizing; the result is the maximum between both sizing paths.

The load-shift sizing procedure introduces several additional quantities:

- H_{shed} : the hours in the shed period during which the heater is off.
- H_{lu} : the number of hours in the load-up period (the time window immediately before the shed period), during which the heater pre-heats to a higher setpoint.
- V_{shed} : total demand volume during the shed block, scaled by the demand fraction corresponding to the percentile of days with successful shed the system is being sized for.
- V_{lu} : demand volume during the load-up period.

LOAD-SHIFT GENERATION RATE

The required load-up period generation rate of DHW, G_{lu} , must account for both the DHW consumed during the load-up period, V_{lu} , as well as all the water that must be heated so that the heater will not need to turn on again during the shed period, V_{shift} .

$$G_{lu} = \frac{(V_{shift} + V_{lu})}{H_{lu}}$$

V_{lu} is a straightforward calculation of the DHW load over the course of the load up period from the first hour, h_{lu_0} , to the last hour, $h_{lu_0} + H_{lu}$

$$V_{lu} = \sum_{h=h_{lu_0}}^{h_{lu_0}+H_{lu}} L(h)$$

The V_{shift} calculation must accommodate the amount of hot water already in the storage tank before the load-up period begins. In addition, V_{shift} accounts for the fact that an adequate amount of DHW must be generated to not have the tank turn on rather than just avoid a DHW outage. It is possible that there is still usable water in the tank at the point the shed setpoints would trigger the aquastat fraction to turn on, however, this water is considered inaccessible by the V_{shift} equation as it is designed to prevent the heater from turning on.

$$V_{shift} = V_{shed} * \frac{S(f_{lu,off}, T_{lu,off}) - S(f_{normal,on}, T_{normal,on})}{S(f_{lu,off}, T_{lu,off}) - S(f_{shed,on}, T_{shed,on})}$$

Where $S(f_{lu,off}, T_{lu,off})$ is the stratification factor at the point in time where the heater is triggered off during load-up (fully charged tank), $S(f_{normal,on}, T_{normal,on})$ is the stratification factor at the point in time where the heater is triggered on during a normal period, $S(f_{shed,on}, T_{shed,on})$ is the stratification factor at the point in time where the heater is triggered on during a shed period.

V_{shed} is a straightforward calculation of the DHW load over the course of the load up period from the first hour, h_{shed_0} , to the last hour, $h_{shed_0} + H_{shed}$

$$V_{shed} = CDF(p) * \sum_{h=h_{shed_0}}^{h_{shed_0} + H_{shed}} L(h)$$

Where $CDF(p)$ is the cumulative density function of the user defined percent of load shift covered, p . At 1.0 the system is sized for the maximum (design-day) load. At 0.89, for example, it accepts that the shed will be violated on the most demanding $\approx 11\%$ of days in exchange for a smaller tank.

$S(f_{lu,off}, T_{lu,off}) - S(f_{shed,on}, T_{shed,on})$ is the gallons of T_{supply} temperature water that can be used from a fully charged tank to the point when the heater would trigger on in shed mode, represented as a fraction of total tank volume. To successfully complete a shed period without turning on the heater, V_{shed} must be less than or equal to $(S(f_{lu,off}, T_{lu,off}) - S(f_{shed,on}, T_{shed,on})) * V_{storage}$

$S(f_{lu,off}, T_{lu,off}) - S(f_{normal,on}, T_{normal,on})$ is the gallons of T_{supply} temperature water that needs to be generated to get from the point that the heater is triggered on in normal mode to the point where the tank is fully charged in load up mode (worst case scenario for entering load up mode because the tank needs to fully charge from the point where it is “empty”), represented as a fraction of total tank volume.

The algorithm uses the aquastat setpoints for the ON trigger during shed and the OFF trigger during load up to find the total storage volume needed to adequately complete all shed periods. Define the stratification factor for the tank in a load shifting scenario as:

$$S_{ls} = S(f_{lu,off}, T_{lu,off}) - S(f_{shed,on}, T_{shed,on})$$

The actual LS generation rate used for sizing is $G_{ls} = \max(G, G_{lu})$ where $G = V_d / H$ is the normal generation rate.

LOAD-SHIFT RUNNING VOLUME

Load shift running volume must be at least V_{shed} because that is the amount of DHW consumed during the shed period when the heater cannot generate more DHW. To calculate the volume needed for the entire day, the deficit algorithm accumulates from the point the first shed period ends:

$$V_{ls,run} = V_{shed} + \max\left(0, -\min_{k \geq 0} \left(\sum_{j=0}^k d_{ls}(h_{shed\ end} + j) \right) \right)$$

Where $h_{shed\ end}$ is the end of the last shed hour and $d_{ls}(h) = G_{ls} - CDF(p) \cdot L(h)$ and generation is zero during shed hours. The load-shift storage volume is then:

$$V_{ls,storage} = \frac{V_{ls,run}}{S_{ls}}$$

where $S_{ls} = S(f_{lu,off}, T_{lu,off}) - S(f_{shed,on}, T_{shed,on})$ is the effective stratification denominator for the load-shift band (the supply-temperature capacity difference between the load-up and shed aquastat positions in a 100-gal tank).

1A SIZING FOR SWING TANK SYSTEMS

A swing tank system has two storage volumes: a primary tank (HPWH + stratified hot water storage) and a smaller temperature maintenance (TM) tank, typically heated through electric resistance, that sits on the recirculation loop. The TM tank isolates the primary HPWHs from high recirculation return temperatures allowing them to heat the coldest incoming water – which is especially beneficial for CO2 refrigerant HPWHs.

TM TANK SIZING

The TM tank is sized from the recirculation loss rate alone. The required TM volume is the smallest entry in the TM sizing table [40, 50, 80, 100, 120, 160, 175, 240, 350, 400, 500, 600, 800, 1000, 1250 gal] that satisfies:

$$V_{TM} \geq \frac{Q_{recirc}}{100 \text{ W/gal} \times 3.412 \text{ BTU/hr/W}}$$

The TM element capacity in BTU/hr is:

$$Q_{TM} = k_{TM} \times Q_{recirc}$$

where k_{TM} is the user defined safety factor (default 1.75) and Q_{recirc} is the recirculation heat loss in BTU/hr:

$$Q_{recirc} = f_{recirc} \times 60 \times \rho c_p \times (T_{supply} - T_{ret})$$

with f_{recirc} the loop flow rate (GPM) and T_{ret} the return water temperature.

PRIMARY TANK SIZING — RUNNING VOLUME

The primary HPWHs and storage tanks carry the primary load (incoming city water heated to hot water supply) and a portion of the temperature maintenance load because high temperature water from the primary storage adds heat to the swing tank when hot water is drawn throughout the building. The swing tank outlets to the mixing valve and, because the water in swing tank storage is often at a lower temperature than the $T_{storage}$ water in the primary tank, a larger percentage of flow through the mixing valve often comes through the hot side than a system where the primary storage is connected directly to the mixing valve. The amount of hot water flow out of the swing tank to the mixing valve is correlated to the amount of water from the primary storage that flows into the swing tank, heating it. The running volume is computed by simulating the swing tank from the start of each peak period in the load shape.

The simulation tracks two temperatures — the swing tank temperature (T_{swing}) and the temperature of the water leaving the primary storage ($T_{storage}$, storage temperature) — and determines how much of each hour's demand can be supplied by the swing tank's stored heat vs. how much must come from the primary.

For each start of a peak period h^* (the start of a peak period is the first hour where the DHW demand of the building surpasses the DHW generation rate of the primary system G after a period where G is greater than the DHW demand) in the peak load shape, a 24-hour simulation of the swing tank is run starting from h^* . Let $g_{swing}(t)$ and $T_{swing}(t)$ be respectively the volume (in gallons) and temperature of water leaving the swing tank at minute t to cover DHW usage, replacing that water with $T_{storage}$ temperature water from the primary. The primary sees a demand profile $g_{primary}(t) = g_{swing}(t)$. The running volume at peak h^* is:

$$V_{run}(h^*) = -\min_{k \geq 0} \left(\sum_{t_0=h^*+60}^k [G_{min}(t) \times f_{mix} - g_{primary}(t)] \right)$$

$$V_{run} = \max(V_{run}(h^*))$$

where $G_{min}(t)$ is the effective per-minute generation rate and f_{mix} (the effective mix fraction) is:

$$f_{mix} = \frac{\sum_t g_{swing}(t)}{V_d}$$

The overall running volume is the maximum over all peak transitions. It is then converted from storage-temperature to supply-temperature frame:

$$V_{supply} = V_{run} \times \frac{(T_{storage} - T_{CW})}{(T_{supply} - T_{CW})}$$

The primary storage volume is then derived using the same stratification derate as normal sizing:

$$V_{storage} = V_{supply} / S(f_{on}, T_{on}).$$

PRIMARY TANK CAPACITY — SWING OVERRIDE

The capacity formula for a swing tank system uses storage temperature (not supply temperature) in the ΔT term, and incorporates the effective mix fraction f_{mix} :

$$Q = \frac{V_d \times f_{mix}}{H} \times \rho c_p \times \frac{(T_{storage} - T_{CW})}{D}$$

This is physically consistent because the primary flow into the swing tank is storage temperature $T_{storage}$ (not supply temperature) from the primary storage tanks, and the mix fraction accounts for the fraction of demand satisfied by the primary rather than the swing tank.

1B SIZING FOR PARALLEL LOOP SYSTEMS

A parallel loop system has the recirculation loop running in parallel with the primary HPWH storage. A small TM tank on the recirc loop provides mixing and temperature maintenance.

The capacity formula is identical to the base formula (Section 1.1, Step 1) because the primary system only handles the DHW use load. No separate recirc capacity term is added — unlike Single Pass Return to Primary (SPRTP) systems, because the TM HPWH handles the recirc loop independently.

The running volume and storage sizing algorithm is also the same as the base case (Section 1.1, Step 2). The load shape used is the peak daily load shape for the building type.

TM storage volume, V_{TM} , is sized so the parallel tank has enough thermal mass to absorb steady-state recirc losses during a full off-cycle without the TM element firing. Specifically, the volume is calculated as:

$$V_{TM} = (k_{TM} \times Q_{recirc} / \rho c_p) \times (H_{off} / (T_{TM,off} - T_{TM,on}))$$

Where H_{off} is the user-configured off-cycle duration for the TM heater in hours (default is 20 minutes or 0.33 hours), $T_{TM,off}$ is the temperature off setpoint for the parallel loop heater, and $T_{TM,on}$ is the temperature on setpoint for the parallel loop heater

TM capacity is sized purely as a user-configured safety-factor multiple of the steady-state recirc loss rate: $k_{TM} \times Q_{recirc}$ (default safety factor 1.75). The safety factor ensures the element can outpace the loss and recover temperature rather than just keeping pace with it.

1C SIZING FOR SINGLE PASS RETURN TO PRIMARY (SPRTP) SYSTEMS

In a single pass return to primary (SPRTP) system, recirculation return water flows back directly into the primary storage tank rather than through a separate TM vessel. This means the primary HPWH must heat both the DHW load and the continuous recirculation heat loss. There is no separate TM element.

CAPACITY

The HPWH must offset the full 24-hour recirc loss during its H -hour daily runtime. The recirc contribution is:

$$Q_{recirc,cap} = k_{TM} \times Q_{recirc} \times \frac{24}{H}$$

Total required capacity:

$$Q_{total} = Q_{DHW} + Q_{recirc,cap}$$

where Q_{DHW} is the base DHW capacity from Section 1.1 Step 1. The $(24/H)$ factor arises because the HPWH offsets 24 hours of continuous recirc loss during its H -hour runtime.

RUNNING VOLUME

Because adding the recirculation load to the original load shape reduces peakiness, the running volume is calculated on the original load shape with no recirculation added. The running volume is sized the same as the running volume is sized in a no recirc system, but with different aquastat locations and stratification factors.

LOAD-SHIFT RUNNING VOLUME

During heater-off periods the recirc loop drains heat from storage continuously, beyond what the DHW load shape captures. The load-shift running volume algorithm is extended by adding the 24-hour recirc loss expressed as an equivalent daily gallon demand at supply temperature:

$$V_{recirc,daily} = \frac{k_{TM} \times Q_{recirc} \times 24}{\rho c_p \times (T_{supply} - T_{CW})}$$

This volume is added to V_d and factored into the load shape function $L(h)$ before the deficit algorithm runs, so the generation rate and demand profiles both include the continuous recirc drain. The result is a larger running volume (and thus a larger primary tank) compared to an equivalent parallel-loop system.

The stratification slope used for SP RTP sizing defaults to 1.7 °F per 1%-height increase (vs. 2.8 for systems without return to primary), reflecting the different tank stratification characteristics of SP RTP configurations (see table 2).

1D SIZING FOR MULTI PASS RETURN (MP RTP) TO PRIMARY SYSTEMS

A multi pass return to primary (MP RTP) system routes recirculation return water back to the primary storage system, similar to a SP RTP system. Each pass of water from primary storage is heated incrementally by the HPWH (rather than heated immediately to storage temperature, like in a single-pass system) resulting in a growing “slug” of tepid water which must be heated to at least supply temperature before being delivered to the building. This topology requires a different tank-sizing approach. Instead of a deficit integral over a load-shape profile, the minimum tank volume is found by simulating the “slug” of cold water that accumulates during peak demand periods.

CAPACITY

MP RTP capacity, Q_{HPWH} (in BTU/hr), is sized identically to the SP RTP formula (Section 1c), with one difference: the default maximum daily run hours is $H = 14$ hr (versus 16 hr for SP RTP), reflecting the higher effective heating demand per run hour imposed by the multi-pass topology.

$$Q_{DHW} = \frac{V_d}{H} \times \rho c_p \times \frac{(T_{supply} - T_{CW})}{D}$$

$$Q_{recirc,cap} = k_{TM} \times Q_{recirc} \times \frac{24}{H}$$

$$Q_{HPWH} = Q_{DHW} + Q_{recirc,cap}$$

RUNNING VOLUME — GROWING-SLUG METHOD

The minimum physical tank volume is found by a 2-day, 1-minute-timestep simulation in which the heater is assumed to run continuously at the capacity Q_{HPWH} . The “slug” is a fully-mixed virtual accumulator representing a mixture of cold water, recirculation loop return water, and heated water at the bottom of the primary tank that is incrementally heated more as additional cold and return water enter the tank.

At each 1-minute timestep the following sequence executes:

1. **Mixing-valve draw.** The mixing valve determines how much water is drawn from primary storage, accounting for cold city water, recirculation return flow, and the temperature of water at the outlet of the primary storage tank. This also determines the correlated quantity (V_{draw}) and temperature (T_{draw}) of the water entering the tank from the combined recirculation loop and cold city water draw.

2. **Slug accumulation.** The drawn volume, V_{draw} , at temperature T_{draw} is blended into the slug via a weighted-average energy balance, updating the slug temperature from $T_{slug,current}$ to $T_{slug,mixed}$ and the volume from $V_{slug,current}$ to V_{slug} :

$$T_{slug,mixed} = \frac{(V_{slug,current} \times T_{slug,current} + V_{draw} \times T_{draw})}{V_{slug,current} + V_{draw}}$$

$$V_{slug} = V_{slug,current} + V_{draw}$$

3. **Heater input.** The heater adds heat to the slug each minute:

$$\Delta T_{slug} = \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{Q_{HPWH}}{V_{slug} \times \rho c_p}$$

$$T_{slug} = T_{slug,mixed} + \Delta T_{slug}$$

4. **Slug reset.** If $T_{slug} \geq T_{storage}$ the slug is considered fully reheated and resets to zero volume at the cold-water inlet temperature. The reset threshold is the storage temperature $T_{storage}$ (not T_{supply}) to ensure the tank can heat to setpoint.
5. **Peak tracking.** The maximum slug volume observed across the entire 2-day simulation is the minimum required running volume, V_{run} (gal). Like other sizing methods, the stratification-factor conversion is applied using the systems normal on setpoint

$$V_{run} = \max(V_{slug})$$

$$V_{storage} = V_{run} / S(f_{on}, T_{on})$$

The recirculation return flow passes through the mixing valve calculation at every timestep regardless of DHW demand. Recirculation losses are captured in Q_{HPWH} via the capacity formula; the slug simulation adds only the storage volume needed for the demand-induced cold-water accumulation.

CAPACITY BOOST

The Q_{HPWH} capacity formula may underestimate the true requirement because it does not account for the cycling volume's (see Figure 1) contribution to the slug at the beginning of a heating period. In the original running volume calculation, the size of the slug at the beginning of a heating cycle or storage temperature reset is 0, however, the cycling volume will contribute to the slug's initial size as it will add a volume of cold water to the slug. To account for this, the capacity is upsized after the initial sizing and tank construction. An iterative correction loop runs up to 3 times:

6. **Trial simulation.** Run a 3-day simulation with the current capacity. The tank is initialized at the heater on-trigger fraction.
7. **Outage detection.** If the usable tank volume ever reaches zero, record the minimum tank outlet temperature $T_{min,outlet}$ reached during the outage and the number of elapsed minutes $t_{deficit}$ since the heater last turned on.

8. **Capacity correction.** Compute the energy needed to raise the tank contents from $T_{min,outlet}$ to T_{supply} and spread it over the deficit period:

$$\Delta E = V_{storage} \times p_{useable} \times \rho c_p \times (T_{supply} - T_{min,outlet})$$

$$\Delta Q = \frac{\Delta E}{\frac{t_{deficit}}{60}}$$

where $p_{useable}$ is the usable fraction of the tank (the region above the cold water inlet to the primary storage), $T_{min,outlet}$ is the minimum observed tank outlet temperature during the outage, and $t_{deficit}$ is the duration (in minutes) from the start of the period where the heater was triggered ON to the end of the DHW outage.

9. **Heater update.** Set $Q_{cap} = Q_{HPWH} + \Delta Q$. Rebuild the water heater with the new capacity, Q_{cap} , and repeat. If no outage is detected the loop exits early. If an outage is detected, the capacity is again upsized using the same method.

SIZING CURVE

The MP RTP sizing curve shows the trade-off between heating capacity and tank volume as the daily run hours vary. Unlike other system types, the curve sweeps only downward from the configured 14 hour daily run time, which may be adjusted to a smaller run time by back calculating a new value of H_{back} after the capacity boost (see below).

Each point on the curve is produced by a full sizing run (including the capacity boost loop) at the target run hours. The effective run hours plotted for each point are back-calculated from the final capacity, Q_{cap} (BTU/hr), to account for any boost that was applied:

$$H_{back} = \frac{(V_d \times \rho c_p \times (T_{supply} - T_{CW}) + Q_{TM} \times 24)}{(Q_{cap} \times D)}$$

Where $Q_{TM} = k_{TM} \times Q_{recirc}$, the recirculation loss (BTU/hr) times the temperature maintenance safety factor. Points with non-strictly-decreasing back-calculated run hours are discarded (the capacity boost has stalled and further points are not meaningful), and the sweep stops when the target run hours fall below 9 hours.

LOAD-SHIFT SIZING

MP RTP does not support load-shift sizing. Load-shifting is not recommended for MP RTP systems.

1E SIZING FOR SUPPLEMENTARY HEATING SYSTEMS

Supplementary heating sizing sizes an additional water heater to compensate for an undersized existing DHW system. This supplemental heating includes natural gas, electric resistance, and other heating equipment used to provide trim heating for a heat pump system. There are four supported types of supplementary heating

configurations in Ecosizer: Single Pass Return to Primary in Series (SPRTP in Series), Multi Pass Return to Primary in Series (MPRTP in Series), High Capacity Swing Tank, and Single Pass Return to Primary in Parallel (SPRTP in Parallel). The sizing algorithm for both “in Series” configurations and the High Capacity Swing Tank are very similar to each other.

For all supplementary heating systems, the user must input the storage and heating capacity of the undersized heat pump system which requires supplementary heating. This means the user must first size using one of the Electric Heating Configurations and use the results to propose an undersized primary storage and heating capacity.

Because standard sizing methods are conservative, edge cases allow supplemental heating simulations to pass without added capacity or storage. When these edge cases occur, the supplemental heating simulation uses the standard sizing method to calculate the added capacity required and outputs the minimum required storage.

IN SERIES AND HIGH CAPACITY SWING TANK SIZING

To size the volume and capacity for in series and high capacity swing tanks, Ecosizer first runs a simulation of the undersized base system (being Swing Tank for High Capacity Swing Tank, MPRTP for MPRTP in Series, and SPRTP for SPRTP in Series) from user input primary capacity and storage volume paired with the building the system serves. For every minute, t , of a 48-hour simulation, the amount of water being drawn from the primary system, as well as the temperature deficit below T_{supply} are tracked as $V_{draw}(t)$ and $\Delta T_{deficit}(t)$ respectively.

$$\Delta T_{deficit}(t) = \max(T_{supply} - T_{outlet}(t), 0)$$

Where $T_{outlet}(t)$ is the temperature of water delivered by the undersized base DHW system at minute t of the simulation. Once the simulation has run, if the base system turns out to already meet demand (outage ≤ 10 min and deficit $\leq 2^\circ\text{F}$), the sizing stops, indicating that no supplementary heating is needed. Otherwise, the required supplemental heating is sized based on the worst outage window across those two 2-day arrays, scoring each window by the sum of $\Delta T_{deficit}(t) * V_{draw}(t)$ for every minute t in the window. Let minute w to $w + t_{window}$ be the worst deficit window found in the simulation, where t_{window} is the size of the window in minutes (default to 30 minutes) such that

$$\sum_{t=w}^{w+t_{window}} \Delta T_{deficit}(t) * V_{draw}(t) = \max \left(\sum_{t=i}^{i+t_{window}} \Delta T_{deficit}(t) * V_{draw}(t) \right)$$

Where $i \in \{0, 1, 2, \dots, 48 \text{ hours} * 60 \text{ minutes} - t_{window}\}$. Once this worst-case window is found, the “in Series” capacity, Q_{IS} , and storage volume, V_{IS} , are

$$V_{IS} = R(t_{window}) * \frac{t_{window}}{60 \text{ minutes}} * \sum_w^{w+t_{window}} V_{draw}(t)$$

$$Q_{IS} = \frac{V_{IS}}{t_{window}} * \frac{\sum_w^{w+t_{window}} \Delta T_{deficit}(t)}{t_{window}} * \rho c_p * \frac{60 \text{ minutes}}{1 \text{ hr}}$$

Where $R(t_{window})$ is a constant scalar derived from the ASHRAE diversity ratios in the 2015 ASHRAE Handbook Table 50.15-7. Possible values of t_{window} and $R(t_{window})$ for Ecosizer are in the table below:

t_{window}	$R(t_{window})$
5	0.146
15	0.354
30	0.604
60	1.0

Table 3. Finding $R(t_{window})$ from t_{window} .

The sizing curve is made by using this method at each option for $t_{window} \in \{5, 15, 30, 60\}$

SINGLE PASS RETURN TO PRIMARY IN PARALLEL SIZING

Sizing for SP RTP in parallel works much the same way as sizing for High Capacity Swing Tank and “in Series” models but has a fixed storage volume rather than a variable tank volume. Unlike the other supplementary heating configurations where supplemental heating is applied to a secondary tank downstream of the primary system, SP RTP in Parallel adds additional heating to the primary tank itself. The storage volume the parallel heater works with, $V_{parallel}$, is the portion of that shared tank sits above the parallel heater’s aquastat on-trigger point.

$$V_{parallel} = V_{storage} * S_{parallel}$$

Where $S_{parallel} = S(f_{parallel,on}, T_{parallel,on})$ is the stratification factor at the point where the parallel supplementary heater would turn on (see section 1.2 Stratification Factor) using its ON aquastat fraction, $f_{parallel,on}$, and temperature setpoint, $T_{parallel,on}$.

To size the parallel heater capacity, $Q_{parallel}$, Ecosizer first runs a 3-day simulation of the undersized primary alone and determines if the primary system is already an adequate size (the system is already adequate if total DHW outage ≤ 10 minutes and the temperature deficit from supply temperature is $\leq 2^\circ\text{F}$), if so, no parallel supplementary heater is needed. Otherwise, $L_{parallel}(h)$, the hourly residual load shape that $Q_{parallel}$ will be sized from is

$$L_{parallel}(h) = L(h) - G_{primary}(h) + (safety_factor * G_{recirc})$$

Where $G_{primary}(h)$ is the primary system’s generation rate in gallons of water at supply temperature at every hour h and G_{recirc} is the hourly recirculation loss converted to gallons of supply temperature water.

$$G_{recirc} = \frac{Q_{recirc}}{\rho C_p * (T_{supply} - T_{CW})}$$

$G_{primary}(h)$ is typically the amount of cold water that can be heated to supply temperature in an hour with the user input primary heating capacity, except for hours in shed mode, for which $G_{primary}(h)$ is set to 0.0, indicating the primary heater is off. Subtracting $G_{primary}(h)$ from the peak load shape’s hourly demand,

$L(h)$, gives a signed residual that the supplementary heater must cover. Sizing is then a reverse application of Normal Sizing (section 1.1): instead of "given a generation rate, find the required storage," it solves "given the fixed storage already available ($V_{parallel}$), find the minimum generation rate", checking every possible start hour and window length for the additional rate needed to keep that window's cumulative deficit within the available storage.

The residual is tiled across two days so that a deficit window can wrap past midnight:

$$L_{parallel}(h + 24) = L_{parallel}(h)$$

$$h \in \{0 \dots 23\}$$

The required parallel generation rate, $G_{parallel}$, in gallons of supply temperature water per hour, is the largest rate demanded by any window:

$$G_{parallel} = \max \left(0, \frac{1}{24} \sum_{h=0}^{23} L_{parallel}(h), \max_{\substack{0 \leq i \leq 23 \\ 1 \leq w \leq 48-i}} \frac{\left(\sum_{k=i}^{i+w-1} L_{parallel}(k) \right) - V_{parallel}}{w} \right)$$

The three terms are:

- **Zero.** The rate cannot be negative. If the primary covers every window on its own, no supplementary capacity is required.
- **The daily average floor.** If the residual over a full 24 hours is positive, the primary is undersized in aggregate rather than only at peak, and no amount of storage will recover the deficit. The parallel heater must make up the daily average regardless of window timing.
- **The worst window.** For a window starting at hour i and running w hours, $\sum_{k=i}^{i+w-1} L_{parallel}(k)$ is the cumulative deficit over that window. Subtracting $V_{parallel}$ credits the storage already available, and dividing by w converts the remainder to the constant rate needed to cover it. The binding case is the maximum over every start hour and window length.

The rate is then converted back to a heating capacity in BTU/hr:

$$Q_{parallel} = G_{parallel} \cdot \rho c_p \cdot (T_{supply} - T_{CW})$$

$Q_{parallel}$ defines a single supplementary water heater. Unlike the in-series configurations, no supplementary storage tank is created, since the parallel heater draws on the primary tank volume already accounted for in $V_{parallel}$.

2 SIMULATION

The 3-day design-day simulation models system performance at 1-minute time steps over three consecutive peak-demand days. An annual simulation runs at 10-minute intervals for a full year. The simulation is driven by the building type, size, and weather data (if applicable for equipment performance map and incoming water temperatures). These provide per-timestep demand, outdoor air temperature, and inlet water temperature. The storage tank state is updated each step via heating and draw operations.

2.1 STORAGE TANK MODEL

The primary storage is modelled as a stratified tank. The stratification is represented linearly, as described in previous sections. Once the tank temperature on the linear stratification reaches $T_{storage}$, all water above that point on the tank is also $T_{storage}$ (see Figure 2). Conversely, when water on the stratification line reaches T_{CW} , all water below that point is also T_{CW} . Two operations have the potential to update the tank state each timestep:

- **Heat:** the HPWH injects energy at the top of the tank. Hot water at $T_{storage}$ displaces cooler water downward.
- **Draw:** demand removes hot water from the top of the tank and introduces cold inlet water at the bottom.

The usable volume reported at each timestep is the number of supply-temperature gallons currently available in the tank, computed by integrating from top of the tank to the point where water is at the temperature T_{supply} and finding the cumulative temperature-weighted volume. If this volume is larger or equal to the demand from the building at that time step, the appropriate amount of water will be drawn to accommodate the building. If there is less hot water in the tank than the building demands, the physical DHW demand at T_{supply} will be drawn from the storage tank, regardless of the temperature of water in the tank, and a DHW outage is recorded.

2.2 CONTROLS AND HEATER ON/OFF LOGIC

Each water heater carries a Controls object for each operating mode ('normal', 'loadUp', 'shed'). The Controls object specifies:

- f_{on} : fractional tank height of the ON sensor (0 = bottom, 1 = top).
- T_{on} : temperature at which the heater turns ON.
- f_{off} : fractional tank height of the OFF sensor.
- T_{off} : temperature at which the heater turns OFF.
- T_{outlet} : maximum outlet water temperature for this mode.

At each timestep the heater's on/off state is updated based on the current tank temperature at the sensor heights. Let $T(f)$ be the current temperature at fractional tank height f :

- If OFF and $T(f_{on}) \leq T_{on} \rightarrow$ turn ON.
- If ON and $T(f_{off}) \geq T_{off} \rightarrow$ turn OFF.

The active mode is determined by the control schedule (a 24-element list mapping each hour to ‘normal’, ‘load up’, or ‘shed’). Each mode has its own set of fractional height and temperature setpoints, resulting in different behaviors. For example, a ‘load up’ mode should have $f_{load\ up,on}$ and $f_{load\ up,off}$ very low in the tank to trigger heating until the tank is nearly full of hot water while ‘shed’ mode should have $f_{shed,on}$ and $f_{shed,off}$ high in the tank to only trigger heating when the tank is nearly empty of hot water. Exact values of the temperature and fractional setpoints are specified by user inputs.

2.3 SIMULATION STEP SEQUENCE

Each 1-minute (or 10-minute) timestep executes in the following order:

1. Query the building/climate for DHW demand, outdoor air temperature (OAT), and inlet water temperature for the current interval.
2. Determine the active control mode for the current hour of day.
3. Update each water heater’s on/off state by reading the tank temperature at its f_{on} or f_{off} (as applicable) sensor positions for the current mode.
4. Sum heating output (BTU/hr) and electrical power draw (kW) from all active heaters. Output heating is a function of OAT and top-of-tank temperature via the heater’s performance map (if running a simulation on a specified make and model) or its nominal capacity from sizing or user inputs.
5. Apply heating to the water in the storage tank.
6. Draw DHW demand from the top of the storage tank, replacing it with cold inlet water (mixed with returning recirculation water if applicable to the system configuration).
7. Record per-timestep metrics: DHW demand from the building, usable volume of water still in storage, heater output for the interval, power intake for the interval, OAT, inlet temperature, and tank temperature profile (the temperature of the tank at each 20% tank height increment up the tank).

2A SWING TANK SIMULATION

The swing tank simulation (both regular and high capacity) adds a second storage tank — the swing tank — that interacts with the recirculation loop and partially buffers demand before the primary storage is tapped. The swing tank differs from the primary storage tank model in that the entire swing tank storage is assumed to be the same temperature rather than having a stratified gradient.

The simulation loop proceeds as follows each timestep:

- ☐ The recirculation loop continuously draws heat from the swing tank at the loop’s return temperature. The swing tank’s electric resistance TM element turns on when the tank temperature drops below the TM on-setpoint ($T_{swing\ on}$), turning off at $T_{swing\ off}$.
- ☐ Domestic hot water demand is drawn first from the swing tank (which replaces stored heat with primary feed water).

- The primary tank sees the thermal feed to the swing tank. The draw from the swing tank to meet building demand is the exact draw in physical gallons from primary storage, which replaces that drawn water with T_{CW} temperature water

Because the swing tank absorbs part of the demand, the primary HPWH cycles less frequently. The usable volume reported at each step is the sum of supply-temperature or above gallons in the primary tank. However, if the swing tank delivers water at a temperature below T_{supply} , the usable volume will be reported as zero since the swing tank must be kept at or above T_{supply} in order to supply DHW to the building.

2B PARALLEL LOOP TANK SIMULATION

The parallel loop simulation follows the base simulation sequence (Section 2.3) with the addition of a TM tank on the recirculation loop. The TM tank is a small, fully-mixed tank (one temperature throughout, like the storage of the swing tank model) whose electric resistance element maintains the recirc loop temperature.

At each timestep the recirculation loop draws water from the TM tank at the pump speed of the recirculation loop in gallons per minute. Recirculation return water entering the TM tank mixes with its stored volume. The TM element turns on when the TM tank temperature drops below $T_{TM\ on}$ and shuts off at $T_{TM\ off}$. The TM tank state is tracked independently from the primary stratified tank. Usable volume is reported for the primary tank only.

2C SINGLE PASS RTP (SPRTP) SIMULATION

The SPRTP simulation is similar to the normal simulation described in section 2.3, however, in addition to the storage tank losing heat to water being drawn, it also loses heat to recirculation loss at every time step. The method performs a two-step energy-to-thermocline conversion. First it computes the heat energy extracted from the tank by the recirc loop during the timestep:

$$Q_{recirc_step} = V_{recirc_step} \times \rho c_p \times (T_{supply} - T_{ret})$$

Where V_{recirc_step} is the gallons of T_{ret} (returning recirculation loop temperature) water that returns to the system during the time step. Then it converts that energy loss into an equivalent thermocline shift in gallons — asking "how many gallons of the current hot-to-cold temperature range carries this much energy?":

$$\Delta V = Q_{recirc_step} / (\rho c_p \times (T_{storage} - T_{CW}))$$

The stratification profile in the storage tank is then shifted down by ΔV as that is the equivalent amount of $T_{storage}$ temperature water being replaced with T_{CW} temperature water as the shift in tank energy from recirculation loss.

2D MULTI PASS RTP (MPRTP) SIMULATION

In the MPRTP simulation the recirculation return enters the bottom of the primary tank directly, mixing with stored hot water. There is no separate TM tank. This has two consequences for the simulation:

- The inlet temperature seen by the storage tank is a blend of cold city water (from DHW demand) and warmer recirculation return water. The effective inlet temperature and amount of water entering the tank each minute is directed by the mixing valve (see Appendix A)
- The HPWH heats the primary tank to offset both DHW load and continuous recirc losses. Because return water enters at a higher temperature than cold city supply, the tank stratification is affected differently than in other systems: the warm return water mixes with cold water being drawn into the tank, potentially giving a slightly longer effective usable volume at supply temperature.

The usable volume is computed the same way as the base simulation — gallons at or above supply temperature in the stratified primary tank. There is no secondary TM volume contribution.

The heater's on/off state is updated via normal aquastat logic. When the heater turns on, the “growing slug” section of the tank is activated, which pulls the sub-supply-temp area out of the base storage tank stratification profile into a fully-mixed “slug” overlay at the bottom of the usable area of the tank. Each minute the heater is on, the water heater adds energy (BTU) directly to that slug, and any drawn volume is removed from the main tank and added to the slug as water of the temperature determined by the mixing valve (see Appendix A). The slug's new size and temperature is determined in the exact same way as the “Growing Slug Method” (section 1d). The slug stays active until the heater turns off, at which point the storage tank merges all slug BTUs back into the stratification energy profile.

While the slug is large enough, the off-sensor sits inside the slug zone, meaning the heater turns off when the slug reaches the off-trigger setpoint.

2E IN SERIES SIMULATION

The simulation for the In Series tank in SP RTP In Series and MP RTP In Series functions much like the Swing Tank simulation in that hot water is pulled to the building occupants from the “in Series” tank (the amount of water pulled is determined by the mixing valve calculation, see Appendix A) which is then replenished by water pulled from primary storage. The difference from the Swing Tank simulation is that the remaining returning water from the recirculation loop enters the primary tank rather than the In Series tank. Thus, from the primary tank's perspective, the simulation is the same as the base model (SP RTP for SP RTP in Series, MP RTP for MP RTP in Series) with a slightly different amount of water being pulled from primary storage as that is now dependent on the temperature of water in the “in Series” tank. The steps of the simulation are as follows:

- Temperature setpoints in the primary system are checked and the primary heater heats its water storage if necessary
- The recirculation loop returning water temperature and flow rate, DHW demand from the building, temperature of cold inlet water, and current temperature of the water in the In Series tank are used in the mixing valve calculation (Appendix A) to determine the physical amount of water in gallons that will be drawn from the In Series tank this time step. The number of gallons drawn from the In Series storage is V_{draw} . The remaining gallons of water returning from the recirculation loop that was not pulled in by the mixing valve to mix with the drawn water is $V_{recirc,remaining}$.

- The average temperature of the top V_{draw} gallons of the primary storage is $T_{draw,primary}$. V_{draw} gallons is then removed from primary storage and replaced with $V_{recirc,remaining}$ gallons of water at recirculation return temperature and $V_{draw} - V_{recirc,remaining}$ gallons of cold inlet temperature water.
- The In Series tank draws in the V_{draw} gallons of $T_{draw,primary}$ temperature water that was in primary storage and recalculates the new temperature of water in the In Series tank.
- The In Series heater heats the water in the In Series storage if directed to do so by its setpoints.

APPENDIX A: SUPPORTING CALCULATIONS

MIXING VALVE INLET AND FLOW TO PRIMARY SYSTEM DURING PEAK DRAW

HPWH Inlet Temperature during peak draw is important because it dictates available HPWH capacity during a peak draw. The illustration and equations below define a system where warm return water from the building splits to either return to the mixing valve or the storage tank. These equations are only relevant when hot water usage is below a specific threshold (critical usage further defined below) so cold water does not supplement warm return water at the mixing valve's cold inlet to satisfy the mixing valve's supply water temperature setpoint.

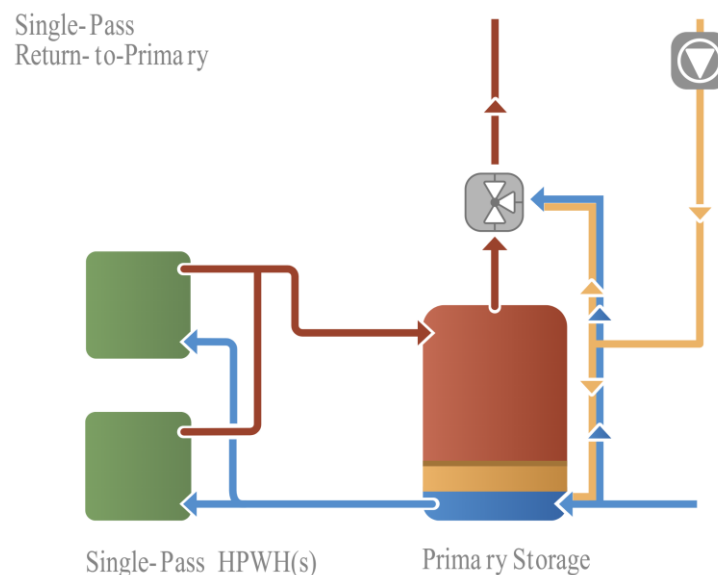


Figure 4. Diagram of Piping around the Mixing Valve .

The Single-Pass Return-to-Primary schematic showcases the mixing valve piping configuration. The Orange line represents the returning recirculation loop water, the blue line represents cold inlet water, and the red line represents hot water. The water leaving the mixing valve (the red line at the very top of the diagram pointing away from the mixing valve) is mixed down to supply temperature for delivery to the building.

1. Assessing the boundaries of the hot water plant, flow entering as cold city (CW) water, at temperature T_i , plus flow entering as hot water return (HWR), at temperature T_{ret} , is equal to flow leaving the mixing valve (hot water supply) at temperature T_{supply} .

$$F_{HWS} = F_{HWR} + F_{CW}$$

- Hot water return flow splits and either flows to the cold mixing valve inlet (F_{HWRMVC}), or the bottom of the tank (F_{HWRT}).

$$F_{HWR} = F_{HWRT} + F_{HWRMVC}$$

- Flow into the hot side of the mixing valve equals flow into the bottom of the tank. This flow consists of all the cold water flow, plus some portion of the hot water return flow.

$$F_{MVH} = F_{CW} + F_{HWRT}$$

- All the water flowing to the cold mixing valve inlet is hot water return.

$$F_{MVC} = F_{HWRMVC}$$

- Mixing ratio equation relates mixing valve flows to temperatures.

$$\frac{F_{MVH}}{F_{MVH} + F_{MVC}} = \frac{T_{supply} - T_{ret}}{T_{storage} - T_{ret}}$$

- Rate of cold water entering the system is equal to the rate of hot water usage.

$$F_{CW} = F_{USAGE}$$

- Hot water return flow can be derived from total heat loss from the recirculation loop and the heat equation.

$$F_{HWR} = \frac{Q_{recirc} * \frac{1 \text{ hr}}{60 \text{ minutes}}}{8.3(T_{supply} - T_{ret})}$$

- The seven equations above are combined to derive tank inlet temperature.

$$T_{TANKIN}$$

$$= \frac{F_{USAGE} \cdot T_{CW}(T_{storage} - T_{ret}) + T_{ret} \left[\frac{Q_{recirc} * \frac{1 \text{ hr}}{60 \text{ minutes}}}{8.3} - F_{USAGE}(T_{storage} - T_{supply}) \right]}{F_{USAGE} \left(T_{supply} - T_{ret} + \frac{Q_{recirc} * \frac{1 \text{ hr}}{60 \text{ minutes}}}{8.3} \right)}$$

CRITICAL HOT WATER USAGE

There is a critical point where there is just enough hot water usage (and associated cold make-up water) that 100% of the primary tank make-up water is cold water, and 100% of the cold mixing valve inlet is warm return water. Setting $F_{HWRT} = 0$ in the equations above defines this case, and results in the following equation:

$$CRITICAL \text{ USAGE} = \frac{Q_{recirc} * \frac{1 \text{ hr}}{60 \text{ minutes}}}{8.3 \cdot (T_{storage} - T_{supply})}$$

If hot water usage is above the critical usage threshold, all warm return water flows through the mixing valve cold inlet and the water temperature entering the tank is equal to cold water temperature.